Roll No. .... Total Pages : 4

GSM/D-20 **892**

PARTIAL DIFFERENTIAL EQUATIONS

Paper - BM-232

Time allowed : 3 Hours *Maximum Marks : 26*

Note : Attempt any five questions, selecting at least one question from each unit. Question No. 1 is compulsory.

Compulsory Question

1. (i) Form the partial differential equation by eliminating the arbitrary constants from the relation $z = ax + by + ab$. 1

(ii) Find the complete integral of

$z = px + qy + \log(pq)$. 1

(iii) Solve :

$(D^2 - D'^2 + D - D')z = e^{2x+3y}$. 2

(iv) Classify the partial differential equation : 1

$$\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = 0.$$

(v) Write one dimensional and two dimensional heat equations. 1

UNIT-I

2. (i) Obtain partial differential equation by eliminating the arbitrary function from

$z = e^{ax-by} f(ax + by).$ 2½

(ii) Solve : $xzp + yzq = xy.$ 2½

3. (i) Find the complete integral of the following equation by using Charpit's Method:

$(p^2 + q^2)x = pz.$ 2½

(ii) Find the complete integral of the following equation by using Jacobi's Method:

$p_1x_1 + p_2x_2 = p_3^2.$ 2½

UNIT-II

4. (i) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y.$ 2½

(ii) Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3.$ 2½

5. (i) Solve $(D^2 - DD' - 2D)z = \sin(3x + 4y).$ 2½

(ii) Solve $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = x^2 y.$ 2½

UNIT-III

6. (i) Classify and reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

to its canonical form. 2½

(ii) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial y \partial x} + \frac{\partial^2 z}{\partial y^2} = 0$$

to its canonical form and hence solve it. 2½

7. (i) Classify and reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0, \mathrm{x} \neq 0,$$

to its canonical form. 2½

(ii) Solve $2s + (rt - s^2) = 1$ by using Monge's method. 2½

UNIT-IV

8. (i) Find the real characteristics of the partial differential equation :

$$y\frac{\partial^2 z}{\partial x^2} + (x + y)\frac{\partial^2 z}{\partial x \partial y} + x\frac{\partial^2 z}{\partial y^2} = 0$$

(ii) Solve the Cauchy problem described by the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

subjected to the initial conditions

$$z(x, 0) = f(x) \text{ and } \left[\frac{\partial z}{\partial y}\right]_{y=0} = g(x).$$ 2½

9. Find the solution of

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ subjected to the boundary conditions

$u(0, t) = u(a, t) = 0, \quad t > 0$

with initial conditions

$u(x, 0) = f(x) \qquad 0 \leq x \leq a,$

and $\frac{\partial u}{\partial t} = (g)x,$ when $t = 0$.