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GSE/D-20 **753**

SOLID GEOMETRY

Paper–BM-113

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all, selecting *one* question from each unit. Question No. 1 is compulsory.

Compulsory Question

1. (a) Find the asymptote of the hyperbola

$$6x^2 - 7xy - 3y^2 - 2x - 8y - 6 = 0.$$ 1

(b) One end of the diameter of the sphere

$$x^2 + y^2 + z^2 - 3x - 2y + 2z - 15 = 0$$

is at the point (–1, 4, 3). Find the co-ordinates of the other end. 1

(c) Find the equation of the cone with vertex at the origin and generators touching the sphere

$$x^2 + y^2 + z^2 - 2x + 4z = 1.$$ 1

(d) Show that the plane $x + 2y + 3z = 2$ touches the conicoid $x^2 - 2y^2 + 3z^2 = 2$ and find the point of contact. 1

(e) Find the equation of tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at a point α. 1

UNIT–I

2. Find the centre, lengths and the equations of the axes, eccentricity, foci and directrices of the conic

$$x^2 + 12xy - 4y^2 - 6x + 4y + 9 = 0.$$ 5½

3. (a) Prove that the conics $x^2 - y^2 - 4x + 2y + 2 = 0$ and $x^2 + 3y^2 - 4x - 6y + 4 = 0$ are confocal. 3

(b) Prove that the conics $x^2 + 3y^2 - 1 = 0$ and

$$2x^2 + 12xy + 39y^2 - 2x - 12y = 0$$

have double contact with each other. Find the co-ordinates of the points of intersection of the tangents at the two points of contact. 2½

UNIT–II

4. (a) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$. 3

(b) Find the equation of the cone whose vertex is the point (–1, 1, 2) and whose guiding curve is $3x^2 - y^2 = 1$, $z = 0$. 2½

5. (a) Find the equation of the right circular cylinder of radius 3 and axis as the line $\frac{x-1}{2} = \frac{y}{2} = \frac{z-3}{1}$. 3

(b) Find the equations of the sphere having the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ as a great circle. 2½

UNIT–III

6. (a) Find the equations of the tangent planes to the surface $x^2 - 2y^2 + 3z^2 = 2$ which are parallel to the plane $x - 2y + 3z = 0$. 3

(b) Prove that the six normals from a point to an ellipsoid lie on a curve of second degree. 2½

7. (a) Show that equations of the polar of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ w.r.t. the quadric $x^2 - 2y^2 + 3z^2 = 4$ are $\frac{x+6}{3} = \frac{y-2}{3} = z - 2$. 3

(b) Find the equation of the enveloping cylinder of the conicoid $2x^2 + y^2 + 3z^2 = 1$ whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{2}$. 2½

UNIT–IV

8. Reduce to the standard form

$2x^2 + 5y^2 + 2z^2 - 2yz + 4zx - 2xy + 14x - 16y + 14z + 26 = 0$

and state the nature of surface represented by the equation. 5½

9. (a) Find the length of semi-axis of the sections of the paraboloid $2x^2 + y^2 - z = 0$ by the plane $x + 2y + z = 4$.

(b) Show that the two confocal paraboloids cut everywhere at right angles. 2½
