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GSE/D-20 **752**

CALCULUS

Paper–BM-112

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *five* questions in all, selecting *one* question from each section. Question No. 1 is compulsory.

Compulsory Question

1. (a) Evaluate $\underset{n\to 0}{\text{Lt}} \frac{x}{|x|}$. 1

(b) If $y = ae^{mx} + be^{-mx}$, prove that $y_2 - m^2 y = 0$. 2

(c) Derive the reduction formula for $\int \tan^n x\, dx$. 2

(d) Define quadrature. 1

SECTION–I

2. (a) Prove that the function f defined as

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous at 0 but not derivable at 0. 2½

(b) Find the nth derivative of $\sin^2 x \cos^2 x$. 2½

3. (a) State and prove Maclaurin's theorem with Lagrange's form of remainder after n-terms. 2½

(b) Expand $\sin x$ in powers of x and hence find $\sin 18°$ upto four decimal places. 2½

SECTION–II

4. (a) Find all the asymptotes of the curve
$x(y - x)^2 - x(y - x) = 2$. 2½

(b) Find the asymptotes of the curve $r = \dfrac{2a}{1 + 2\cos\theta}$. 2½

5. (a) Show that for the curve

$s = \sqrt{8ay}$ $\quad P = 4a\sqrt{1 - \dfrac{y}{2a}}$. 2½

(b) Find the co-ordinates of centre of curvature at any point on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $x = a\cos\theta$, $y = b\sin\theta$. Hence find the equation of its evolute. 2½

SECTION–III

6. (a) Trace the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$. 2½

(b) If $u_n = \displaystyle\int_0^{\pi/4} \tan^n x\, dx$, show that $u_n + u_{n-2} = \dfrac{1}{n-1}$, hence evaluate u_5. 2½

7. (a) Prove that $\int_0^{\pi/2} \sin^{2n} x dx = \frac{2n!}{[2^n(n!)]^2} \cdot \frac{\pi}{2}$. 2½

(b) Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to an extremity of the latus rectum. 2½

SECTION–IV

8. (a) Find the area between the curve $y^2 = \frac{x^3}{2a - x}$ and its asymptotes. 2½

(b) Find the area common to the circle $x^2 + y^2 = 4$ and the ellipse $x^2 + 4y^2 = 9$. 2½

9. (a) Find the volume of the solid formed by the revolution about the x-axis of the curve $y^2(a + x) = x^2(3a - x)$. 2½

(b) Find the centroid of the semi-circular region of radius r by Pappus theorem. 2½