Roll No. .... Total Pages : 4

GSE/M-20 **1450**

MATHEMATICS

(Vector Calculus)

Paper : BM-123

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section.

Compulsory Question

1. (a) Find the volume of a parallelopiped whose edges are represented by $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$. 1

(b) If the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are coplanar, then show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$. 1

(c) Find a so that the vector

$$\vec{f} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$$

is irrotational. 1

(d) Show that div (curl $\vec{f}$) = 0. 1

(e) Determine the transformation from cylindrical to rectangular co-ordinates. 1

SECTION–I

2. (a) Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar. 2½

(b) The necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable t to have a constant magnitude is $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$. 3

3. (a) Show that $\vec{a} \times (\vec{b} \times \vec{c})$, $\vec{b} \times (\vec{c} \times \vec{a})$ and $\vec{c} \times (\vec{a} \times \vec{b})$ are coplanar. 2½

(b) The necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable t to have constant direction is $\vec{f} \times \frac{d\vec{f}}{dt} = 0$. 3

SECTION–II

4. (a) For any vector $\vec{a}$, show that $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$, where $\vec{r}$ is the position vector of a point. Hence show that grad $[\vec{r}\ \vec{a}\ \vec{b}] = \vec{a} \times \vec{b}$. 2½

(b) Prove that $\nabla^2 [r\,\vec{r}] = \left(\frac{4}{r}\right)\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, and $|\vec{r}| = r$. 3

5. (a) If div $(\phi(r)\vec{r}) = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, and $|\vec{r}| = r$, then prove that $\phi(r) = \dfrac{c}{r^3}$. 2½

(b) Prove that $\nabla^2\left[\dfrac{x}{r^2}\right] = -\dfrac{2x}{r^4}$. 3

SECTION–III

6. (a) Express the vector $\vec{A} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates. Hence determine A_ρ, A_θ and A_z. 2½

(b) If (r, θ, ϕ) are spherical co-ordinates, show that $\nabla\left(\dfrac{1}{r}\right) = \nabla \times (\cos\theta\, \nabla\phi)$. 3

7. (a) Transform the function $\vec{f} = P\hat{e}_\rho + P\hat{e}_\phi$ from cylindrical to cartesian co-ordinates. 2½

(b) Express the velocity $\vec{v}$ and accleration $\vec{a}$ of a particle in cylindrical co-ordinates. 3

SECTION–IV

8. (a) Evaluate the line integral $\int_C \vec{f} \cdot d\vec{r}$ about the traingle whose vertices are (1, 0), (0, 1) and (–1, 0) where $\vec{f} = y^2\hat{i} - x^2\hat{j}$. 2½

(b) Verify Green's theorem in the plane for $\oint_C (xy + y^2)\, dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 3

9. (a) Evaluate $\iint_S \vec{f} \cdot \hat{n}\, dS$, where $\vec{f} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. 2½

(b) Evaluate $\oint_C f \cdot d\vec{r}$ by Stoke's theorem, where $\vec{f} = y^2\hat{i} + x^2\hat{j} - (x + z)\hat{k}$ and C is the boundary of triangle with vertices at (0, 0, 0) (1, 0, 0) and (1, 1, 0). 3