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GSQ/M-20 **1722**

MATHEMATICS

BM-362

Linear Algebra

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

Compulsory Question

1. (a) What can you say about the linear span of the empty set ? **1**

(b) Let $T : U \to V$ be a homomorphism, then prove that ker T is a subspace of U. **1**

(c) Show that $T : R^2 \to R^3$ given by **1**

$T(x, y) = (2x - y, x - y, -2x)$ is a linear transformation.

(d) Normalize the vector $u = (2, 1, -1)$ in $\mathbf{R}^3$. **1**

(e) Show that the linear transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + z, x - z, y)$ is non-singular. **2**

Section I

2. (a) Prove that a minimal generating set of a finitely generated vector space V(F) is always a basis of V. **2½**

(b) Show that the union $W_1 \cup W_2$ of subspaces of a vector space V need not be a subspace of V. **2½**

3. (a) Let U = L (S_1) and V = $L(S_2)$, where :

$S_1 = \{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$,

$S_2 = \{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$.

Find basis and dimension of U + V. **2½**

(b) Let W be a subspace of a finite dimensional vector space V(F), then show that $\dim \frac{V}{W} = \dim V - \dim W$. **2½**

Section II

4. (a) Find a linear transformation $T : R^3 \to R^2$ such that $T(1, 1, 1) = (1, 0)$ and $T(1, 1, 2) = (1, -1)$. **2½**

(b) If $T : U \to V$ be a linear transformation, then show that $\dim(R(T)) + \dim(N(T)) = \dim U$. **2½**

5. (a) Find a linear transformation $T : R^3 \to R^3$ whose range is generated by (1, 0, –1), (1, 2, 2). **2½**

(b) If B = {(–1, 1 1), (1, –1, 1), (1, 1, –1)} be a basis of R^3, then find the dual basis of B. **2½**

Section III

6. (a) Let T : $R^3 \rightarrow R^3$ be a linear operator defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}. **2½**

(b) Find the linear map T : $R^2 \rightarrow R^3$ whose matrix is

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 3 \\ 0 & 1 \end{bmatrix},$$ relative to the ordered basis

B = {(1, 1), (0, 2)} and B′ = {(0, 1, 1), (1, 0, 1), (1, 1, 0)} for R^3. **2½**

7. (a) Find the co-ordinates of (1, 2, 1) relative to the basis {(1, 1, 2), (2, 2, 1), (1, 2, 2)} using change of basis matrix (transition matrix). **2½**

(b) If T be an invertible operator and λ is an eigen value of T, then show that λ^{-1} is an eigen value of T^{-1}. **2½**

Section IV

8. (a) Let V be an inner product space, then show that $\|u+v\| = \|u\| + \|v\|$. **2½**

(b) Obtain an orthonormal basis with respect to standard inner product for the subspace of R^3 generated by (1, 0, 1), (1, 0, –1) and (0, 3, 4). **2½**

9. (a) Show that a linear operator T on a unitary space V in Hermitian iff $< T(\alpha), \alpha >$ is real for every α. **2½**

(b) Let T be a linear operator on an inner product space V(F). If $T^2(u) = 0$ and T is self-adjoint or skew-symmetric, then show that $T(u) = 0$. **2½**